XIV INTERNATIONAL CONFERENCE - SYSTEM MODELLING and CONTROL
JUNE 27-29 2011, Łódź, Poland

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Wavelet Adaptation Based on Signal Processing Outcome

Abstract. *The problem of wavelet synthesis is a crucial part of wavelet theory. In this paper an overview of wavelet synthesis methods used in the literature up till now is given. It is demonstrated that these well known approaches suffer from some major drawbacks. A new approach to adaptive wavelet synthesis that overcomes these drawbacks and has not been exploited before is proposed. This approach is based on adapting wavelet to one particular signal and specific signal processing algorithm.*

Keywords: wavelets, wavelet synthesis, signal processing

Introduction

Discrete Wavelet Transform (DWT) became one of the most important tools in the field of signal processing during the last two decades. Unlike the well known Discrete Fourier Transform (DFT) and the Discrete Cosine Transform (DCT), DWT doesn't have one strictly defined set of basis functions. Instead a number of general conditions is imposed on the basis functions. Unbounded degrees of freedom are used to adapt wavelet properties to a desired signal processing application.

In this paper a brief history of the wavelet theory as well as an overview of methods used for wavelet synthesis is presented. Drawbacks of currently used approaches are pointed out. A new concept of wavelet synthesis is proposed to overcome these problems.

Wavelet synthesis methods

Beginnings of the modern wavelet theory date back to the middle eighties of the 20th century. In that time the signal processing researchers were extensively developing the concept of Quadrature Mirror Filter (QMF) banks proposed around 1976-77 [1, 2]. A QMF bank consists of a pair[1] of filters ensuring Perfect Reconstruction (PR) of a signal [9, 10]. Frequency responses of such a pair of filters were designed to split the input signal into two signals with non-overlapping spectra. A notable influence on the development of QMF theory was by Vaidyanathan. In [7] he introduced a concept of a lattice structure[2] for implementation of Finite Impulse Response (FIR) filters. He also demonstrated that lattice structures (LS) are an efficient method of parametrization and implementation of QMF banks. In [11] Vaidyanathan and Hoang demonstrated that lattice structures possess PR property regardless of parameters quantization, which makes it a good choice for optimization purposes. As an example Vaidyanathan and Hoang constructed objective function that led to synthesis of filters attenuating selected frequency ranges. This allowed to construct a filter bank consisting of low-pass and high-pass filters. DFP was used as an optimization method. An additional algorithm for accurate estimation of initial solution was employed to ensure convergence.

At the time when the QMF theory was being developed, first works considered to be the beginnings of a modern wavelet theory have been published [12, 13, 14], however it were the works of Ingrid Daubechies [15,16,17,18] and Stéphane Mallat [19,20,21] that constituted a real breakthrough and provided a solid foundation to the theory. The relations between wavelets and QMF banks were quickly discovered: every scaling function and its corresponding wavelet constitutes a QMF bank, however the converse is not always true [22, 23, 24, 25]. The created theory described a linear transform that differed significantly from the widespread and well known DFT and DCT transforms. In opposition to these transforms, which basis functions are sine and cosine having an infinite support, the DWT used functions with finite support. This allowed to precisely localize signal singularities not only in the frequency domain but also in the time domain. Moreover, the theory did not provide just one set of basis functions. Instead, the conditions regarding basis functions' orthogonality (or biorthogonality), their norm and frequency responses were given. Imposing such conditions left some degrees of freedom unbounded, which implies that the number of possible DWT basis functions is infinite. Daubechies' approach to wavelet synthesis was bounding the remaining degrees of freedom by introducing additional constraints. Thus the Daubechies wavelet family with maximal number of vanishing moments and the Coiflet wavelet family with maximal symmetry[3] were created. These families became the basis for wavelet adaptation to some of the researchers.

Soon the problem of wavelet synthesis became one of the key problems in practical applications of the wavelet theory. Most researchers concentrated their efforts on constructing wavelets ensuring optimal representation of signals, mostly images. The problem of optimal signal representation was considered by Tewfik et al. [6]. They demonstrated methods for constructing wavelets minimizing approximation error for a given scale (number of signal decomposition levels). The problem was also approached by Gopinath et al. [4]. They tackled two problems: what is the optimal wavelet multiresolution of a signal at a given scale and what is the optimal multiresolution of a class of signals. The first problem is the same as the one investigated by Tewfik et al., however Gopinath et al. point out that Tewfik's approach generates suboptimal solutions. Gopinath et al. consider a general case of M-channel filter bank. They notice that the problem of minimizing the approximation error for a given signal depends only on the coefficients of a filter. In order to perform numerical optimization authors use Householder parametrization [26] and BFGS optimization method.

Desarte et al. [22] considered a problem of wavelet optimization for an optimal image coding. They used lattice

[1] Discussion on filter banks consisting of more than two filters may be found in [3, 4, 5, 6, 7, 8].

[2] Terms „lattice filter" and „lattice structure" are commonly accepted and widespread in the signal processing terminology. The term „lattice" comes from the shape of filter connection graph.

[3] Perfect symmetry is impossible for orthogonal wavelets.

structure as a parametrization method and minimization of low-pass filter's output variance was proposed as an optimization criterion. This led to synthesis of a filter bank composed of low-pass and high-pass filters. The problem of choosing optimal wavelet for image representation was also researched by Unser [24]. He remarks that minimizing the approximation error is equivalent to maximizing the energy of the low-pass filter output. Unser points out that such an approach makes sense only when global quantization strategy is applied prior to coding.

Antonini et al. [27] analysed the influence of wavelet's frequency response on its performance in image compression and coding[4]. They conclude that the number of vanishing moments and its regularity (smoothness) are of crucial importance to the quality of the compressed image, however Veterelli and Herley [31] consider the influence of wavelet smoothness on its performance in image compression to be an open question. The problem was also considered by Villasenor et al. [25]. They constructed a general framework for evaluation of filter performance in the task of image compression and used it to compare different wavelets proposed in the literature. The wavelet analysis/synthesis bank was considered to be a linear system. The high-pass wavelet coefficients were zeroed before the image reconstruction. Authors studied the impulse and step responses of such a system and based on them they were able to draw general conclusions on the wavelet's efficiency in image compression. They notice that the Hölder exponent reflecting the smoothness of a wavelet is not sufficient to ensure the wavelet's good performance in image compression. They also point out that the synthesis filter has greater influence on the quality of reconstructed image than the analysis filter and that filters with impulse response of even length handle point singularities better[5]. Another important remark they make is that performance of a filter in 1D signal processing cannot be used to fully characterize the performance of to 2D image compression because of the interactions between vertical and horizontal filter responses.

It must be noted that evaluation of filters in image compression is still an active field of research. A good example is a linear regression model proposed in 2010 by Gaofeng et al. [32]. It allows to accurately predict the quality of a wavelet-compressed image without performing the actual quantization, coding and inverse wavelet transform. The only required information are average image brightness and standard deviation, energy concentration of a wavelet and wavelet coefficients entropy. Authors conclude that the key to good performance in image compression is the wavelet's ability to concentrate energy. Therefore the proposed model can be considered as an extension of the previously used approach based only on the energy compaction [24].

Despite questionable effectiveness of Hölder exponent as a measure of wavelet effectiveness it was later used by some of the researchers. An example is a work by Lang and Heller [33] who demonstrated that by sacrificing vanishing moments of Daubechies family wavelets and applying numerical optimization one can obtain functions with better Hölder smoothness. The concept of sacrificing vanishing moments was also used by Odegard and Burrus [34]. They set the vanishing moments of low order to zero, while the remaining degrees of freedom were used to minimize a large number of higher order moments. This approach led to creating wavelets allowing better approximation of higher order polynomials than the Daubechies wavelet. In another paper [35] Odegard and Burrus demonstrate a method of biorthogonal wavelet synthesis for image compression based on minimizing the Discrete Finite Variation [36], which is a generalization of Hölder smoothness for discrete signals. Such generalization is based on remark that in practical applications only the finite number of signal analysis levels is taken into account when considering smoothness.

Wei et al. [37, 38] presented a generalized Coiflet family obtained by removing the zero-centered vanishing moment condition. The obtained degrees of freedom allowed to improve the filter's characteristics: symmetry and phase response.

Rieder et al. [23] demonstrated an effective orthogonal wavelet implementation based on lattice structures. Using LS to perform the DWT required imposing additional constraint, which results directly from the previously mentioned dependency that not every QMF bank is a wavelet filter. LS representation used by Rieder et al. ensured the implemented filter is always a wavelet. Authors considered different numerical optimization criteria: compact support, regularity, frequency behaviour (like the previous researchers they also conclude that it's necessary to minimize the energy of the high-pass filter output) and symmetry. Their implementation was based on CORDIC algorithm [59] which allowed to significantly reduce the number of arithmetic operations required to compute the wavelet transform. Authors also emphasize the fact that in practical applications wavelet smoothness is important only for a finite number of analysis level.

Although lattice structures gained notable popularity as a method of wavelet parametrization, other parametrizations have been proposed as well, e.g. [40,41,42,43]. Shark and Yu [44] used poliphase parametrization [26] of a filter bank and applied genetic algorithm to synthesize shift-invariant wavelets. Regensburger [45] introduced parametrization of compactly supported orthonormal wavelets by discrete moments. He presented explicit parametrizations for wavelets with support of length 4, 6, 8 and 10. Lipiński and Yatsymirskyy [46] introduced parametrizations of Daubechies 4 and Daubechies 6 wavelets, demonstrated a construction of transform with real-valued coefficients and showed that their approach allows to reduce the number of arithmetic operations necessary to compute the transform. In [47] Yatsymirskyy introduced a new approach to lattice structures. He demonstrated that it is possible to construct fast neural network with architecture based on the LS and use it for adaptive wavelet synthesis. Another method of wavelet parametrization and implementation is the lifting scheme [48,49]. It allows to implement biorthogonal wavelets [50], integer-to-integer wavelet transform [51] and second generation wavelets [52,53].

Wavelet adaptation based on signal processing results

From the above survey it can be concluded that so far two wavelet analysis approaches have been used. The first approach relies on optimizing the scaling or wavelet

[4]The problem of wavelet image compression is a broad subject. Apart from the obvious importance of wavelet filters used for image decomposition, the applied quantization strategy and coding scheme are also very important. Due to specifics of wavelet decomposition of a two dimensional signal, numerous coding schemes of 2D wavelet coefficients have been proposed. The most important are Shapiro's classical Embedded Zero-tree Wavelet (EZW [28]), Set Partitioning In Hierarchical Trees (SPIHT [29]) or the Embedded Block Coding with Optimal Truncation (EBCOT [30]) used in the JPEG2000 standard.

[5]In case of orthogonal wavelets synthesis and analysis filters are identical and have even length. This is not the case with biorthogonal wavelets, which have different analysis and synthesis filters and can have impulse responses of odd length.

function based on some criterion defined by the expert, e.g. smoothness or symmetry. It is assumed that if the given wavelet function will have some arbitrarily chosen property then it will perform good in a selected signal processing application. It must be emphasized that among researchers there is no agreement what kind of wavelet property would definitely ensure optimality in a selected application. It must also be noted that even if such a property could be determined, a wavelet synthesized in this way could only be optimal in a statistical sense for a given class of signals.

The second of the so far used wavelet synthesis approaches is far better: the wavelet is selected in such a way to provide the most accurate approximation of a signal or class of signals[6]. There are two main advantages of such an approach. Firstly, the wavelet may be adapted to a particular signal, not to a whole class of signals, however such a possibility was not recognized and considered important in the literature. Secondly, the arbitrary assumptions about wavelet properties are discarded and replaced by conditions imposed on the properties of the processed signal. Nevertheless, in practical applications, concentration of energy, although important, is just one of the steps in a signal processing chain, e.g. in compression it is followed by quantization and coding. Therefore this approach to wavelet synthesis does not take into account all the characteristics of a signal processing algorithm.

Authors of this paper claim that replacing assumptions about wavelet properties with conditions imposed on the processed signal and adapting wavelet to a particular signal instead of adapting it to a class of signals is an approach with huge potential that has not been utilized so far. It is therefore proposed to synthesize the wavelet function using only conditions imposed on the characteristics of processed signal. This will allow to adjust the wavelet both to the processed signal and signal processing algorithm.

The results obtained by Gaofeng et al. in an already mentioned paper [32] show that proposed approach is promising. Let us remind that Gaofeng et al. concluded that in case of image compression the performance depends on energy concentration of the DWT, characteristics of processed image and characteristics of the DWT processed signal (in this case it was entropy influencing quantization and coding of coefficients). So far this last component was ignored in all the wavelet synthesis methods.

Summary

In this paper it was demonstrated that approaches to wavelet synthesis used so far suffer from serious drawbacks: they either rely on some arbitrary assumptions about wavelet characteristics or ignore the characteristics of signal processing application to which the wavelet will be applied. A new approach was proposed to overcome these problems. It relies on adapting the wavelet to characteristics of both the processed signal and the signal processing algorithm. The only optimality criterion is the final result of signal processing. Proposed approach is being actively developed and has been successfully applied to improve digital image watermarking performance [54].

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